QUANTIFYING SUPPRESSED VARIANCE IN FIXED-AMPLITUDE SIMULATIONS

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Paving the way for next generation of cosmological surveys

Sexten Center for Astrophysics

Gaussian initial conditions

Box of side L, fundamental frequency $k_f = 2\pi/L$.

ICs determined by the initial power spectrum is $P_0(k) = P_L(k)/D^2(z)$, set by fiducial cosmology.

Gaussian density field in Fourier space:

$$\delta^L_{\mathbf{k}} = \sqrt{\frac{P_L(k)}{2k_f^3}} r_{\mathbf{k}} e^{i\theta_{\mathbf{k}}}, \tag{1}$$

with $r_{\mathbf{k}}$ Rayleigh-distributed and $\theta_{\mathbf{k}}$ uniformly distributed in $[0, 2\pi)$.

For a gaussian field, $\left\langle \delta^L_{\mathbf{k}_1} \ldots \delta^L_{\mathbf{k}_N} \right\rangle_c = 0$ for $N > 2$.

Statistics of N-body simulations

N-body simulations are useful for estimating the power spectrum at small-scales.

- A single simulation could have large fluctuations at large scales (cosmic variance), thus introducing a bias for the estimation of power at small scales
- Running a large number of simulations is computationally expensive

Since we are limited by the scatter in $P(k) \sim \langle \delta_{\mathbf{k}} \delta_{-\mathbf{k}} \rangle \sim \langle r_{\mathbf{k}} r_{-\mathbf{k}} \rangle$, we can reduce this scatter by fixing the amplitude $r_{\mathbf{k}}$.

Fixed-amplitude initial conditions

Fixed-amplitude linear density field: $r_{\mathbf{k}} = \sqrt{2}$

$$\delta^L_{\mathbf{k}} = \sqrt{\frac{P_L(k)}{k_f^3}} e^{i\theta_{\mathbf{k}}} \tag{2}$$

with $\theta_{\mathbf{k}}$ uniformly distributed in $[0, 2\pi)$.

First formalized in Pontzen *et al.*, 2016, and Angulo & Pontzen, 2016.

We lose gaussianity of linear field.

All even connected N-point correlation functions are different from zero:

$$\left\langle \delta^L_{\mathbf{k}_1} \delta^L_{\mathbf{k}_2} \delta^L_{\mathbf{k}_3} \delta^L_{\mathbf{k}_4} \right\rangle_c \neq 0 \quad , \quad \left\langle \delta^L_{\mathbf{k}_1} \delta^L_{\mathbf{k}_2} \delta^L_{\mathbf{k}_3} \delta^L_{\mathbf{k}_4} \delta^L_{\mathbf{k}_5} \delta^L_{\mathbf{k}_6} \right\rangle_c \neq 0 \quad , \quad \ldots \tag{3}$$

Are these relevant for the density field?

Quantifying non-gaussianity of fixed-amplitude density field

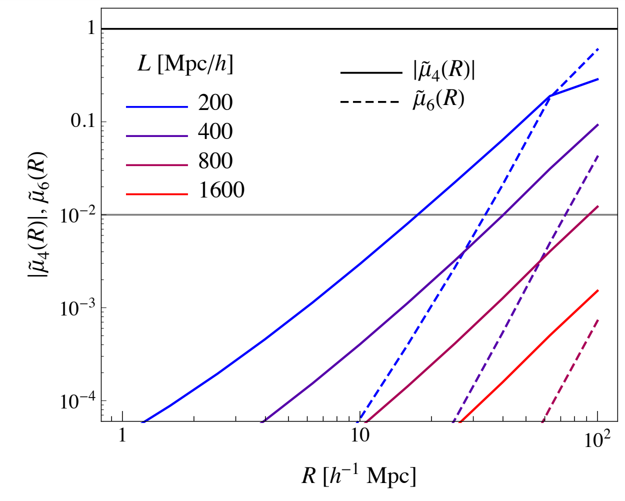
Higher order statistical moments of the fixed-amplitude field smoothed on a scale R:

$$\tilde{\mu}_4(R) = \frac{\mu_4(R)}{\sigma^4(R)} = \left.\frac{\left\langle \delta^4(\mathbf{x}) \right\rangle_c}{\langle \delta^2(\mathbf{x}) \rangle_c^2}\right|_R \propto \frac{1}{V} \frac{\int \mathrm{d}^3\mathbf{k} W^4(kR) P_L^2(k)}{\left[\int \mathrm{d}^3\mathbf{k} W^2(kR) P_L(k)\right]^2} \tag{4a}$$

$$\tilde{\mu}_6(R) = \frac{\mu_6(R)}{\sigma^6(R)} = \left.\frac{\left\langle \delta^6(\mathbf{x}) \right\rangle_c}{\langle \delta^2(\mathbf{x}) \rangle_c^3}\right|_R \propto \frac{1}{V^2} \frac{\int \mathrm{d}^3\mathbf{k} W^6(kR) P_L^3(k)}{\left[\int \mathrm{d}^3\mathbf{k} W^2(kR) P_L(k)\right]^3} \tag{4b}$$

In general, $\tilde{\mu}_{2n}(R) \propto V^{-n}$

Quantifying non-gaussianity of fixed-amplitude density field



[AO *et al.*, preliminary]

Covariance matrix

Power spectrum estimator:

$$\hat{P}(k) = \frac{k_f^3}{N_k} \sum_{\mathbf{q} \in k} \delta_{\mathbf{q}} \delta_{-\mathbf{q}} \qquad \Longrightarrow \qquad P(k) = \left\langle \hat{P}(k) \right\rangle \tag{5}$$

Covariance matrix

$$\begin{aligned} \mathbb{C}_{ij} &= \frac{k_f^6}{N_{k_i} N_{k_j}} \sum_{\mathbf{q} \in k_i} \sum_{\mathbf{p} \in k_j} \left[2 \langle \delta_{\mathbf{q}} \delta_{\mathbf{p}} \rangle \langle \delta_{-\mathbf{q}} \delta_{-\mathbf{p}} \rangle + \langle \delta_{\mathbf{q}} \delta_{-\mathbf{q}} \delta_{\mathbf{p}} \delta_{-\mathbf{p}} \rangle_c \right] = \\ &= \frac{2 \delta_{ij}^K}{N_{k_i}} P^2(k_i) + \frac{k_f^3}{N_{k_i} N_{k_j}} \sum_{\mathbf{q} \in k_i} \sum_{\mathbf{p} \in k_j} T(\mathbf{q}, -\mathbf{q}, \mathbf{p}, -\mathbf{p}) \end{aligned} \tag{6}$$

Covariance in fixed-amplitude initial conditions

With fixed-amplitude ICs, the even N-point correlation functions on the linear fields are non-zero:

$$\left\langle \delta^L_{\mathbf{k}_1} \delta^L_{\mathbf{k}_2} \delta^L_{\mathbf{k}_3} \delta^L_{\mathbf{k}_4} \right\rangle_c = -\frac{P_L^2(k)}{k_f^6} \left[\delta^K_{\mathbf{k}_{12}} \delta^K_{\mathbf{k}_{34}} \delta^K_{\mathbf{k}_{13}} \delta^K_{\mathbf{k}_{24}} + \delta^K_{\mathbf{k}_{12}} \delta^K_{\mathbf{k}_{34}} \delta^K_{\mathbf{k}_{14}} \delta^K_{\mathbf{k}_{23}} + \delta^K_{\mathbf{k}_{13}} \delta^K_{\mathbf{k}_{24}} \delta^K_{\mathbf{k}_{14}} \delta^K_{\mathbf{k}_{23}} \right] \tag{7}$$

Therefore, the covariance becomes:

$$\begin{aligned} \mathbb{C}_{ij} &= \frac{2\delta^K_{ij}}{N_{k_i}} P^2(k_i) + \frac{k_f^3}{N_{k_i} N_{k_j}} \sum_{\mathbf{q}\in k_i} \sum_{\mathbf{p}\in k_j} T(\mathbf{q}, -\mathbf{q}, \mathbf{p}, -\mathbf{p}) = \\ &= \frac{2\delta^K_{ij}}{N_{k_i}} P^2(k_i) + \frac{k_f^3}{N_{k_i} N_{k_j}} \sum_{\mathbf{q}\in k_i} \sum_{\mathbf{p}\in k_j} \left[T_0(\mathbf{q}, -\mathbf{q}, \mathbf{p}, -\mathbf{p}) + T_{gr}(\mathbf{q}, -\mathbf{q}, \mathbf{p}, -\mathbf{p}) \right] = \\ &= \frac{2\delta^K_{ij}}{N_{k_i}} \left[P^2(k_i) - P_L^2(k_i) \right] + \frac{k_f^3}{N_{k_i} N_{k_j}} \sum_{\mathbf{q}\in k_i} \sum_{\mathbf{p}\in k_j} T_{gr}(\mathbf{q}, -\mathbf{q}, \mathbf{p}, -\mathbf{p}) \end{aligned} \tag{8}$$

Motivations

The covariance matrix can be written as

$$\mathbb{C}_{FIX} = \mathbb{C}_{GIC} - \frac{2\delta_{ij}^K}{N_{k_i}} P_L^2(k_i), \tag{9}$$

where $\mathbb{C}_{GIC}$ contains both the term $\sim P^2(k)$ and the term $\sim T_{gr}$; therefore we can recover the "real" covariance matrix with gaussian ICs:

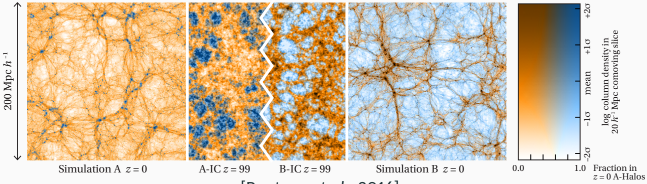
$$\mathbb{C}_{GIC} = \mathbb{C}_{FIX} + \frac{2\delta_{ij}^K}{N_{k_i}} P_L^2(k_i). \tag{10}$$

At a given redshift z, we would expect

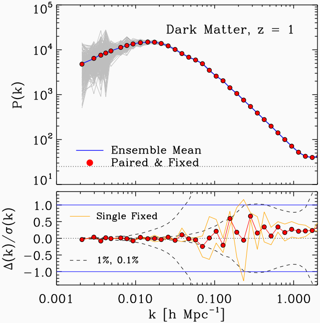
$$\mathbb{C}_{GIC}(z) = \mathbb{C}_{FIX}(z) + \frac{2\delta_{ij}^K}{N_{k_i}} D^2(z) P_L^2(k_i). \tag{11}$$

Pairing fixed-amplitude simulations

Two fixed-amplitude realisations, with **opposite phases**: $\delta^{\uparrow}_{\mathbf{k}}$, $\delta^{\downarrow}_{\mathbf{k}}$. At linear level, $\delta^{\downarrow}_{L,\mathbf{k}} = \delta^{\uparrow}_{L,\mathbf{k}} e^{i\pi} = -\delta^{\uparrow}_{L,\mathbf{k}}$.



[Pontzen *et al.*, 2016]



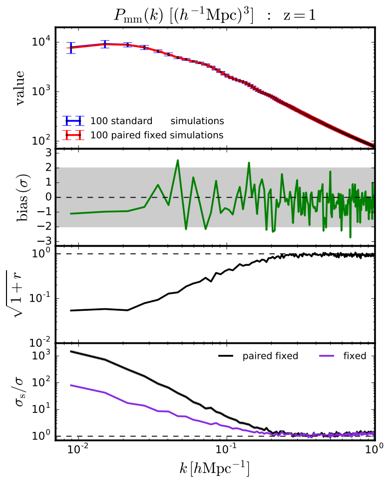
[Angulo & Pontzen, 2016]

Power spectrum:

$$P(k)^{\updownarrow} = \frac{1}{2}\left[P^{\uparrow}(k) + P^{\downarrow}(k)\right] \qquad (12)$$

State of the art

Further extensions in a recent paper (Villaescusa-Navarro *et al.*, 2018)



- Power spectra of matter, halos, CDM, gas, stars, BHs, magnetic fields
- Cross-spectra
- Mass functions of halos, voids
- PDFs of density fields

Covariance matrix in fixed-and-paired initial conditions

Given the power spectrum of the paired realisation $P(k)^{\updownarrow} = \frac{1}{2}\left[P^{\uparrow}(k) + P^{\downarrow}(k)\right]$, all sorts of cancellations arise in the covariance matrix: **further suppression**

Power spectrum covariance of fixed-and-paired realizations:

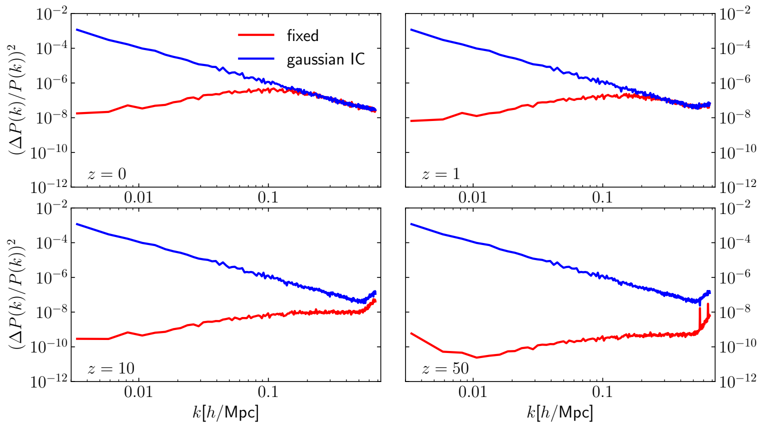
$$\mathbb{C}^{\updownarrow}_{ij} - \mathbb{C}^{\uparrow}_{ij} = -\frac{4\delta^K_{ij}}{N_{k_i}} P^{\uparrow}_L(k_i) P^{\uparrow}_{22}(k_i) \qquad [\mathcal{O}(\delta^6_L)] \tag{13}$$

Comparison with the fixed-amplitude case

$$\begin{aligned} \mathbb{C}^{\uparrow}_{ij} &= \frac{2\delta^K_{ij}}{N_{k_i}} \left[P^2(k_i)^{\uparrow} - P^2_L(k_i)^{\uparrow}\right] = \\ &= \frac{4\delta^K_{ij}}{N_{k_i}} P^{\uparrow}_L(k_i) \left[P^{\uparrow}_{13}(k_i) + P^{\uparrow}_{22}(k_i)\right] \qquad [\mathcal{O}(\delta^6_L)] \end{aligned} \tag{14}$$

Numerical Analysis: fixed-amplitude variance in ZA

[AO *et al.*, preliminary]

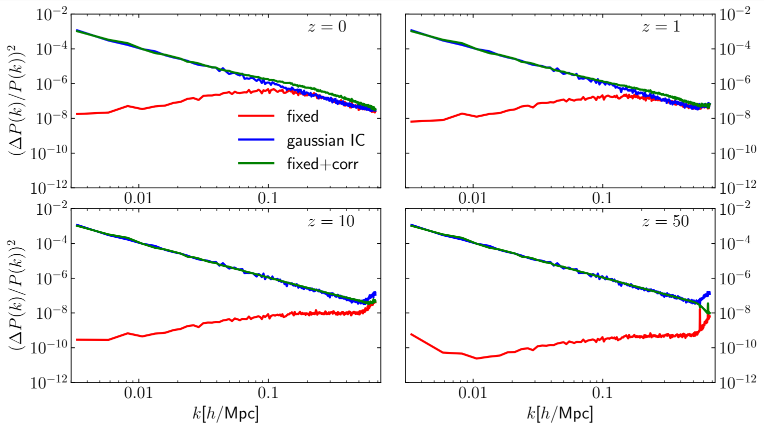


Variance is suppressed, especially at large scales.

Numerical Analysis: fixed-amplitude variance in ZA

$$\Delta P^2_{FIX} = \Delta P^2_{GIC} + \Delta P^2_{NGIC} \quad \Longrightarrow \quad \Delta P^2_{GIC} = \Delta P^2_{FIX} - \Delta P^2_{NGIC}$$

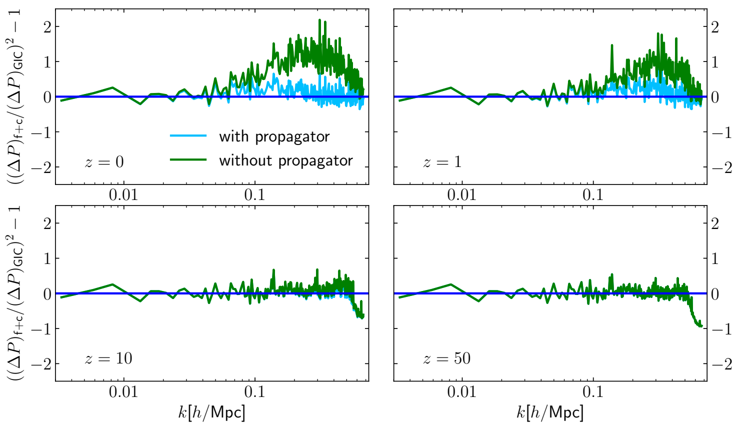
[AO *et al.*, preliminary]



Effect is almost completely under control, except for a “bump”-like feature.

Numerical Analysis: fixed-amplitude variance in ZA

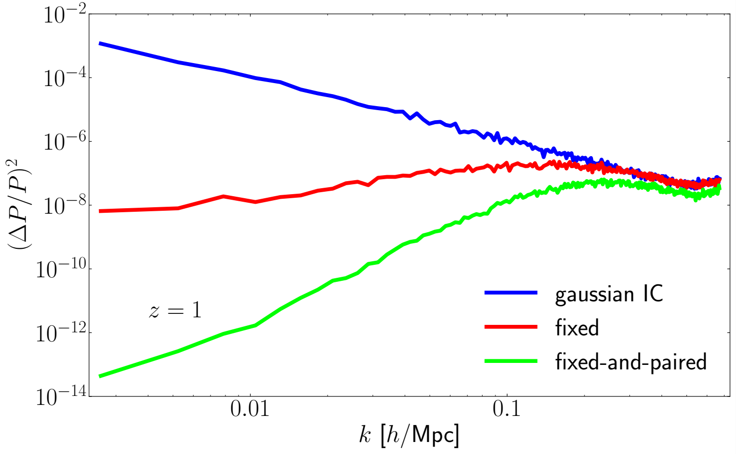
[AO *et al.*, preliminary]



Replacing the linear growth with the non-linear propagator from RPT, the feature is almost canceled.

Numerical Analysis: fixed-and-paired variance in ZA

[AO *et al.*, preliminary]

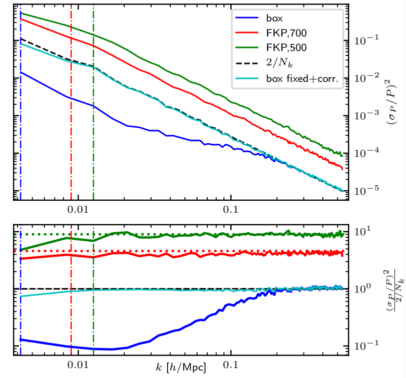


Further variance suppression in the fixed-and-paired realisations.

Numerical Analysis: PINOCCHIO mocks and spherical window function

Set of 10,000 PINOCCHIO mock simulations with gaussian IC, 1,000 mocks with fixed-amplitude, and 1,000 fixed-and-paired.

[AO *et al.*, preliminary]



Similar effects are also visible in the PINOCCHIO mocks.
Suppression is less evident (probably due to the cut-off in halo mass).
Spherical **window function apparently removes the suppression** in the variance (to be investigated).

Conclusions

- Fixing the amplitude of the density field **suppresses the variance**
- The observed suppression is **mostly consistent with the theoretical prediction**
- Pairing fixed-amplitude realisations introduces a **further suppression**
- **Bias** seems to play a role in the suppression
- Introducing a **window function the suppression is practically removed**

Thank you for listening