

Primordial non-Gaussianities:
Zero bias tracers
&
Paving the road to Fisherland

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Sexten CfA, 7/6/2018

An existential crisis

Plenty of data: Future CMB missions + DES, DESI, LSST, Euclid, WFIRST, CHIME, HIRAX

Promise of dramatically improve error bars on cosmological parameters.

However...

Incremental improvement is not enough, not all parameters are born equal.
Precision cosmology means benchmarks to be achieved.

Examples are neutrino masses, inflationary parameters, N_{eff} , curvature, tensor modes...

Dark energy is the elephant in the room in this discussion.

This talk



Primordial non-Gaussianities



**“I suppose I’ll be the one
to mention the elephant in the room.”**

Why we care

Inflation makes a number of testable predictions :

- Observable Universe is flat

$$|\Omega_K| < 0.005$$

- Spectral index and runnings

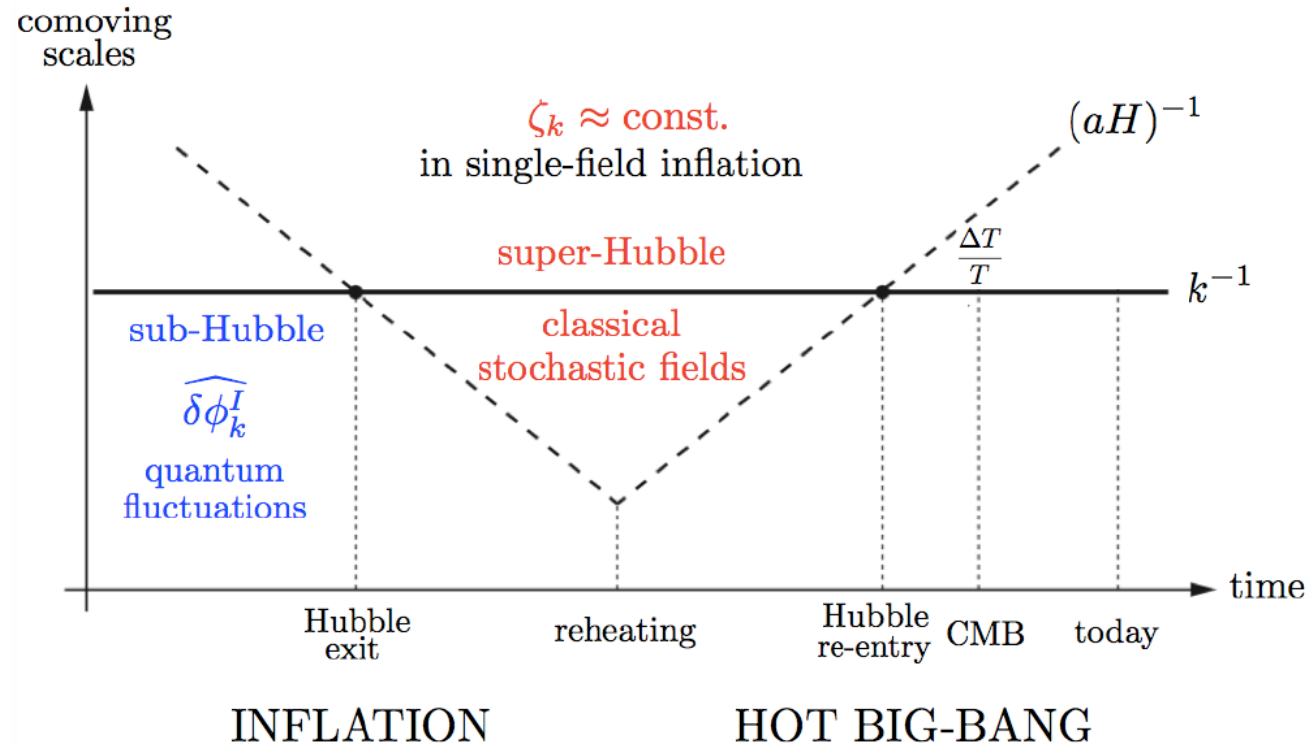
$$n_s = 0.9655 \pm 0.0062$$

- ~ Adiabatic fluctuations

$$\alpha_{\text{iso}} < 1\%$$

- ~ Gaussian fluctuations

- Tensor modes. TBD



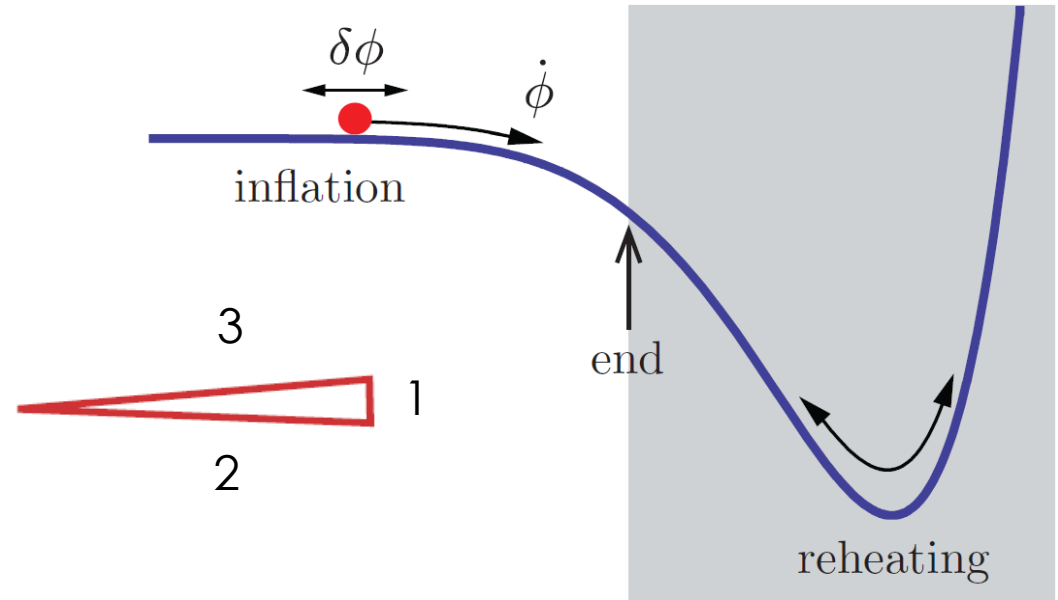
Very general features of inflation, PNG as a tool to make model selection.

The consistency relation

Higher point functions are a useful probe of the dynamics of the inflaton.

Local non-Gaussianities in the curvature perturbations

$$\zeta = \zeta_g + \frac{3}{5} f_{NL} (\zeta_g^2 - \langle \zeta_g^2 \rangle)$$



Credit: D. Baumann

$$\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3)$$

Local non Gaussianities are negligible in single field inflation.
A non perturbative result independent of the dynamics.

$$\lim_{k_1 \rightarrow 0} B_\zeta(k_1, k_2, k_3) = \left[0 + \epsilon \mathcal{O} \left(\frac{k_2}{k_1} \right)^2 \right] P_\zeta(k_1) P_\zeta(k_3)$$

Maldacena
Creminelli&Zaldarriaga

Primordial Non-Gaussianities (PNG)

Detection of local PNG will rule out single field inflation.
Non detection of $f_{\text{NL}} \sim 1$ constrains multi-field models.

If we get there, we are guaranteed to learn something



$$\sigma_{f_{\text{NL}}} \lesssim 1$$

Planck bispectrum measurements yield $f_{\text{NL}} = -0.8 \pm 5$

LSS still far $\sigma_{f_{\text{NL}}} \lesssim 30$

How do we improve on that?

Cosmic Variance

Unique signature

$$P_{gg}(k, \mu, z) = [b + f\mu^2 + f_{NL}(b-1)\alpha(k)]^2 P(k, z) + \frac{1}{\bar{n}(z)}$$

Error bars ~30 now,

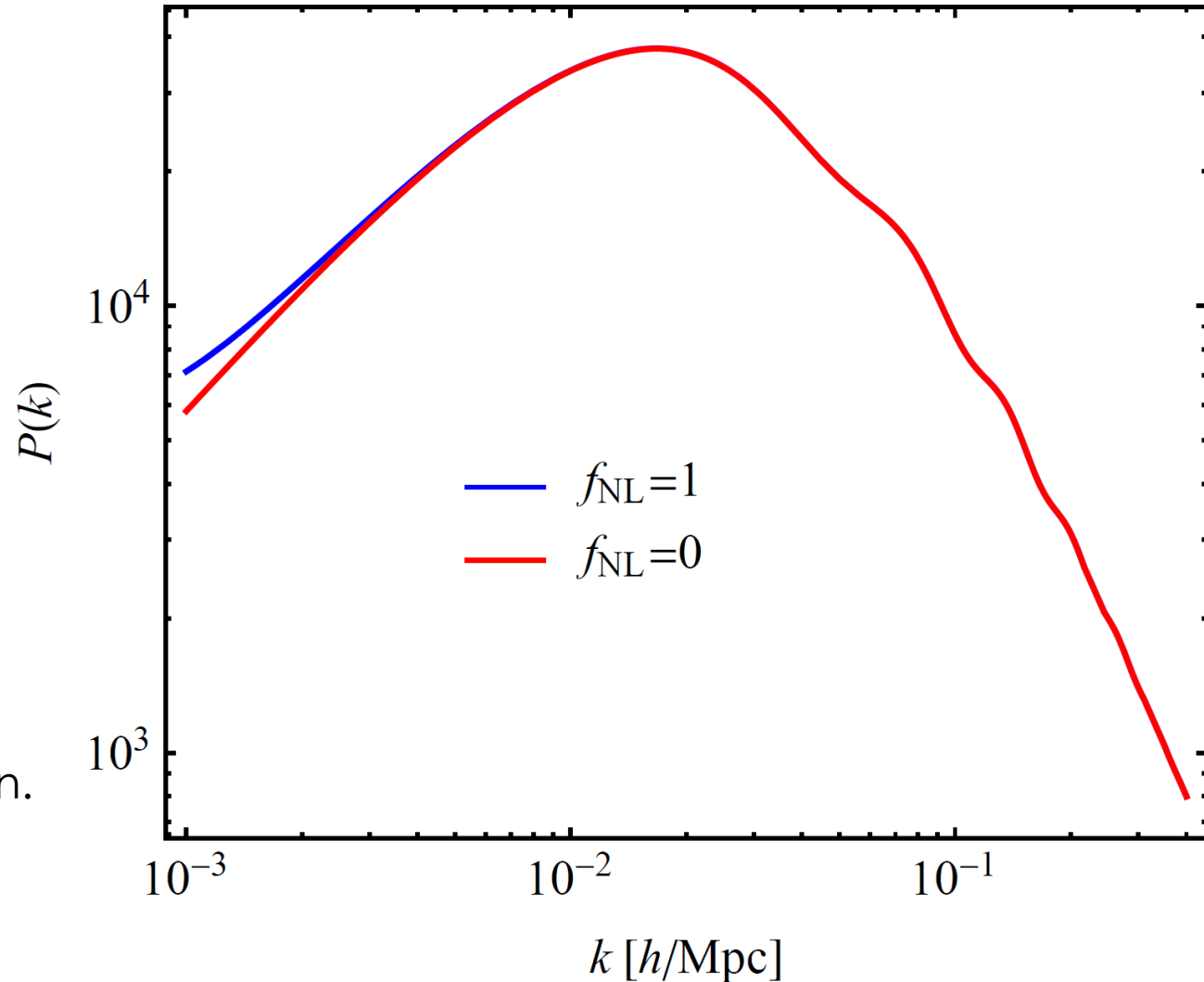
~Comparable to Planck in the future (DESI, LSST)

Two main issues:

- Cosmic Variance is the dominant source of noise.
- Also, systematics at large scales are tough. E.g. Foregrounds, seeing, imaging sys., window function.

Bonus issue:

f_{NL} response is not always $(b-1)$



Cosmic Variance

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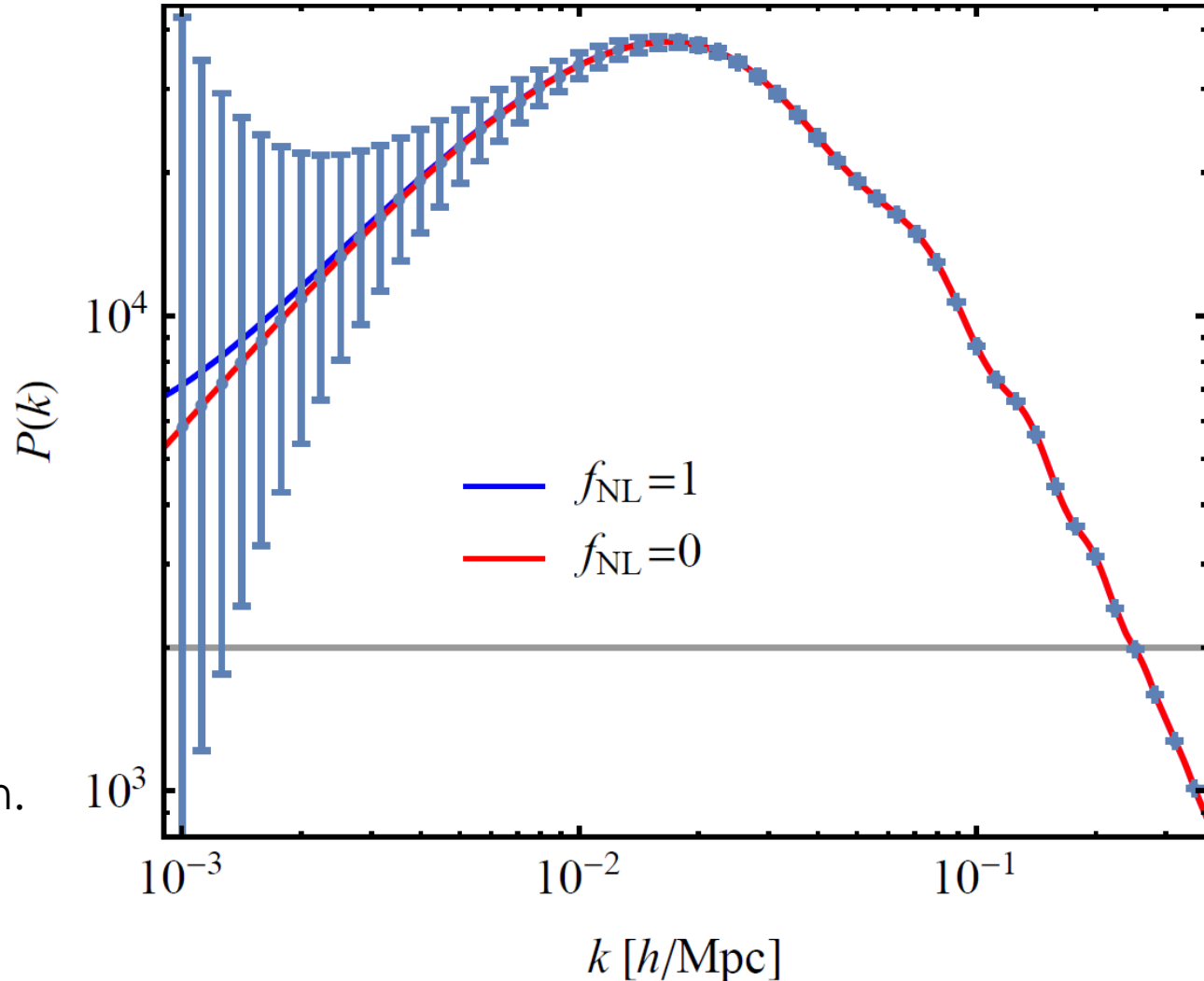
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Cosmic Variance

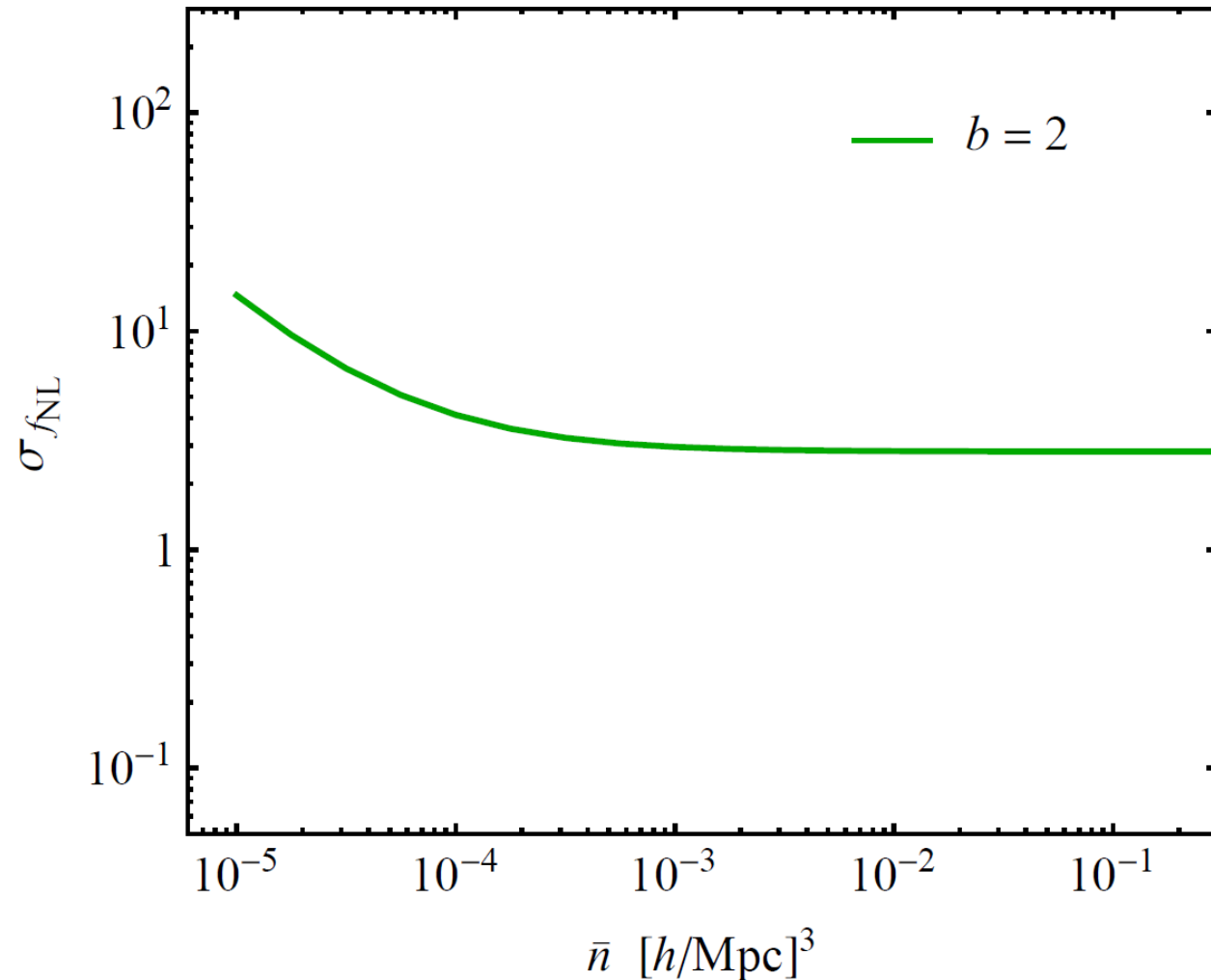
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Cosmic Variance cancellation

In the limit of zero noise sample variance can be canceled

$$\frac{\delta_1}{\delta_2} = \frac{[b_1 + f_{\text{NL}}(b_1 - 1)\alpha(k)]\delta_m + \epsilon_1}{[b_2 + f_{\text{NL}}(b_2 - 1)\alpha(k)]\delta_m + \epsilon_2}$$

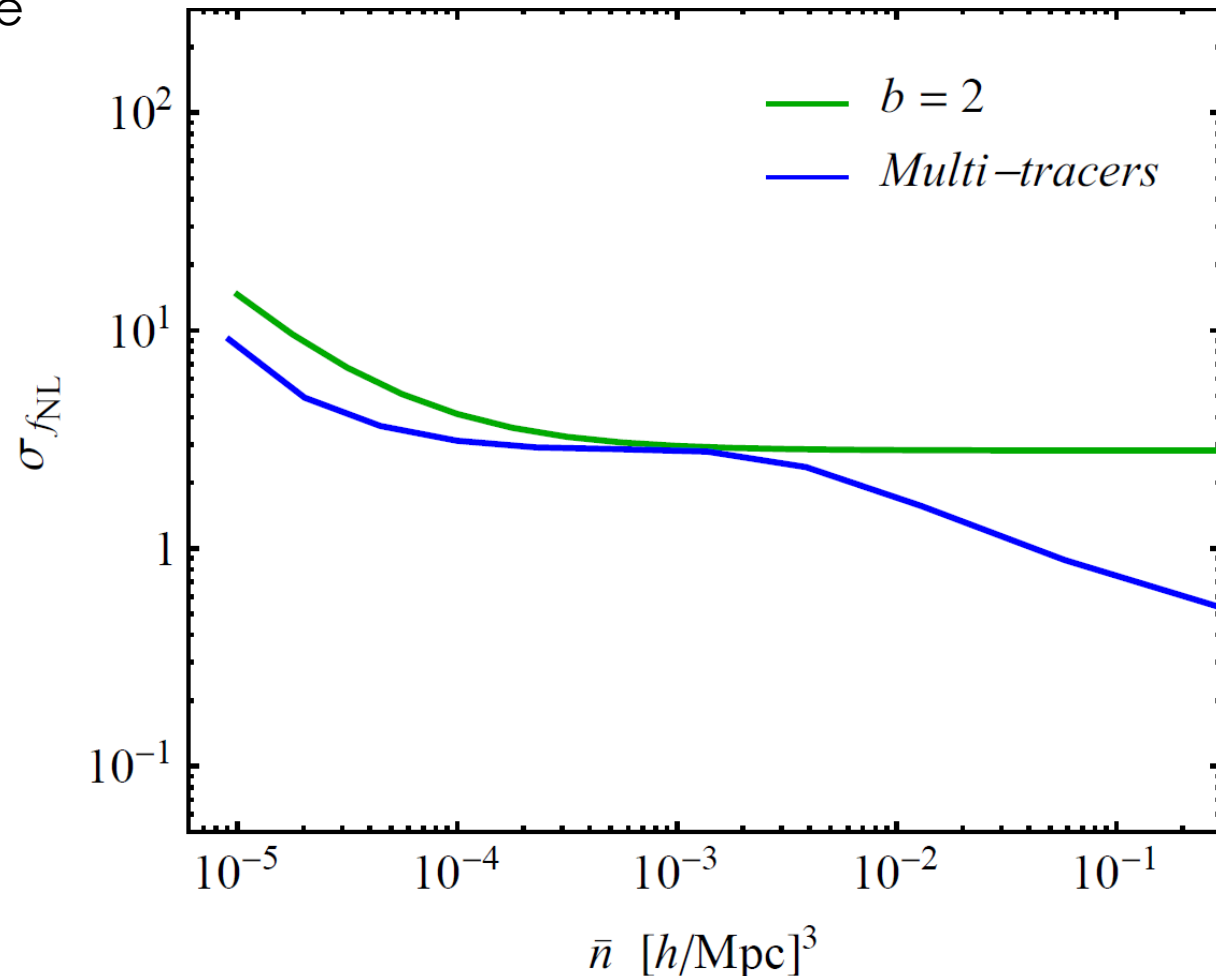
Use cross-correlations! Do not pay the price of CV twice.

Yields large improvements.

Very difficult on real data.
How to split?

CMB as the 2nd tracer.
Schmittfull&Seljak17

Still hard to achieve $f_{\text{NL}} \sim 1$



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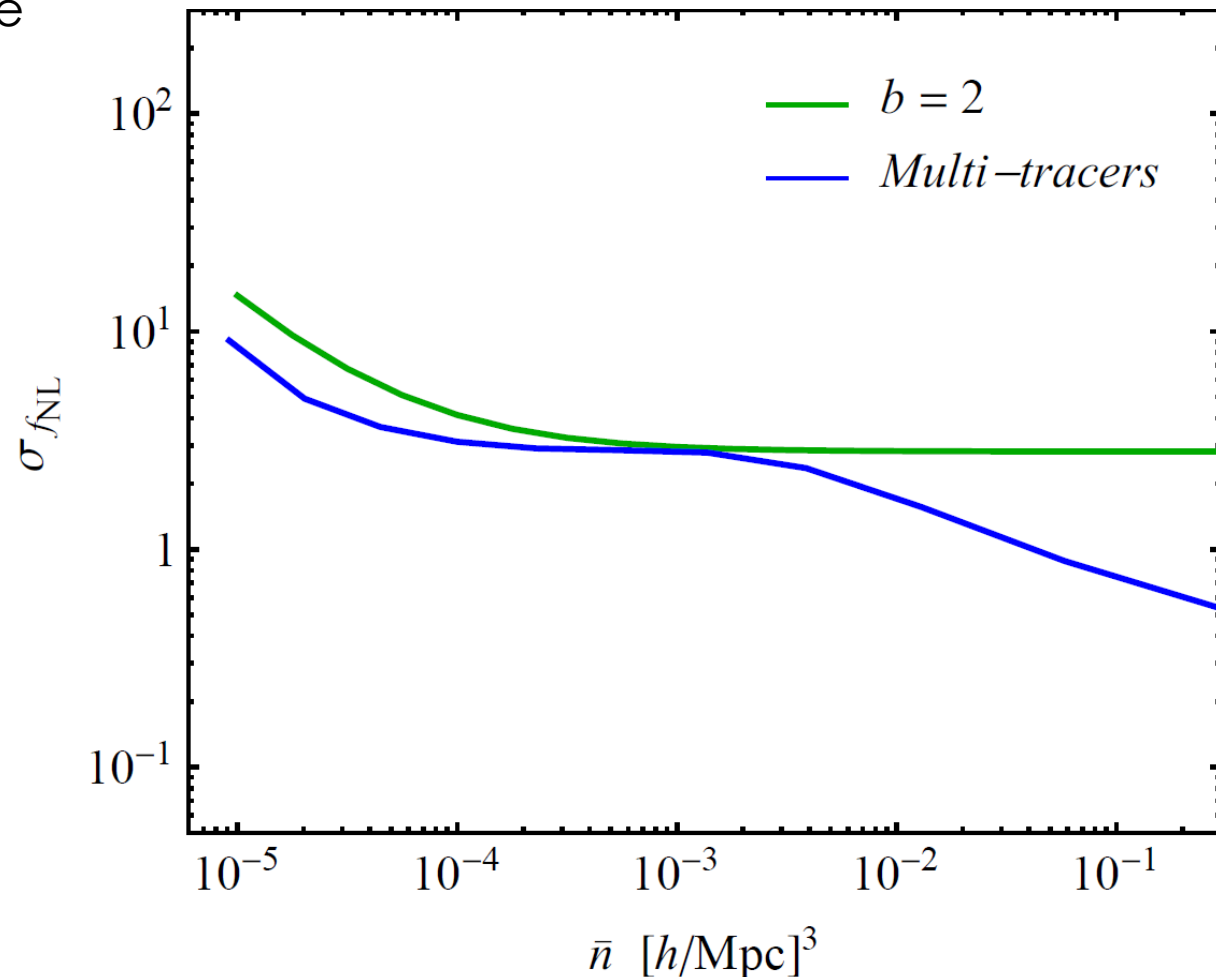
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The real cosmic variance cancellation: zero bias tracers

On large scales the FIDUCIAL power spectrum is

$$\hat{P}_{gg}(k, \mu, z) = P_{gg}(k, \mu, z) + \frac{1}{\bar{n}(z)} = (b + f\mu^2)^2 P(k, z) + \frac{1}{\bar{n}(z)}$$

The error is proportional to the signal...

$$C_{ij} = \langle \hat{P}(k_i) \hat{P}(k_j) \rangle - \langle \hat{P}(k_i) \rangle \langle \hat{P}(k_j) \rangle$$
$$\sigma_P^2 \longrightarrow = 2\delta_{ij} \frac{(2\pi)^3}{N_k} \left(P_{gg}(k_i) + \frac{1}{\bar{n}} \right)^2 + \text{Trispectrum}$$

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The bottom line: If bias is zero Cosmic Variance is zero ! Left with shot noise only.

The real cosmic variance cancellation: zero bias tracers

Fisher information

$$\sigma_{f_{\text{NL}}}^{-2} = F_{f_{\text{NL}} f_{\text{NL}}} \propto \frac{b^2 (b-1)^2 \alpha(k)^2 P^2(k, z)}{\left(b^2 P(k) + \frac{1}{\bar{n}} \right)^2}$$

← Signal

Shot noise dominated regime

↑ CV ↑ Noise

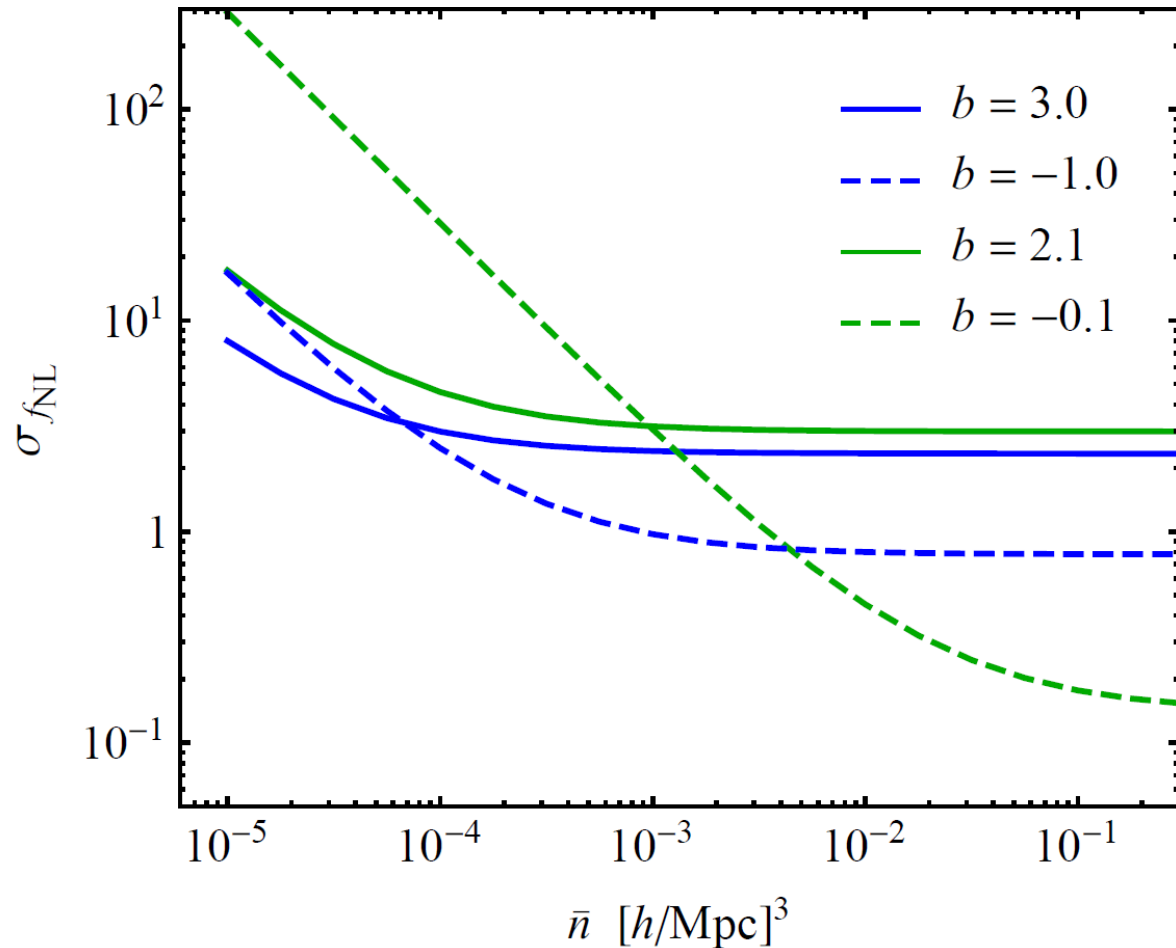
$$F_{f_{\text{NL}} f_{\text{NL}}} \rightarrow b^2 (b-1)^2 \alpha(k)^2 \bar{n}^2 P^2(k, z)$$

CV dominated regime

$$F_{f_{\text{NL}} f_{\text{NL}}} \rightarrow \frac{(b-1)^2 \alpha(k)^2}{b^2}$$

In principle zero bias could achieve infinite precision on fnl.

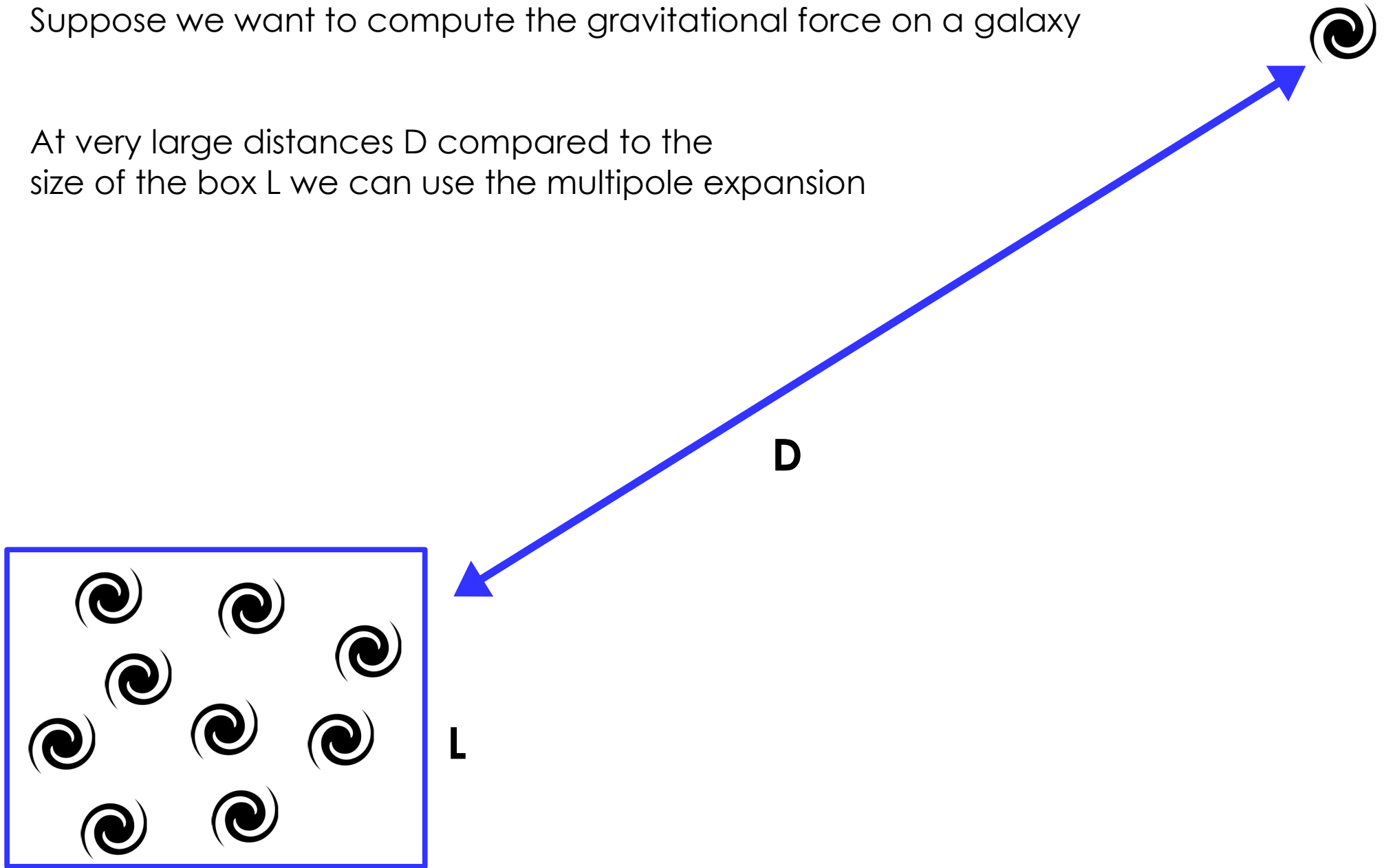
Halos/Galaxies never have zero bias if selected by mass/luminosity.



A zero bias field

Suppose we want to compute the gravitational force on a galaxy

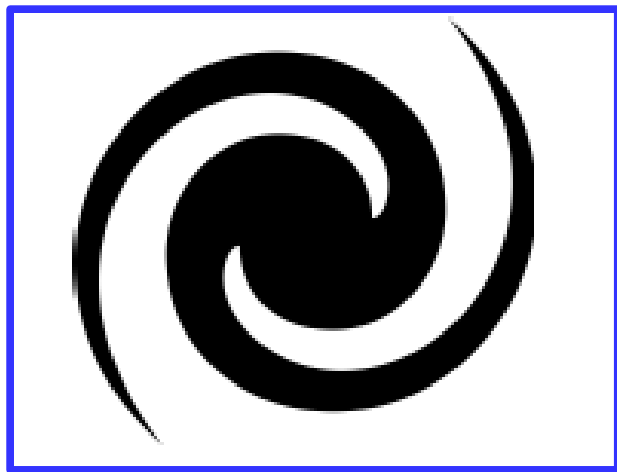
At very large distances D compared to the size of the box L we can use the multipole expansion



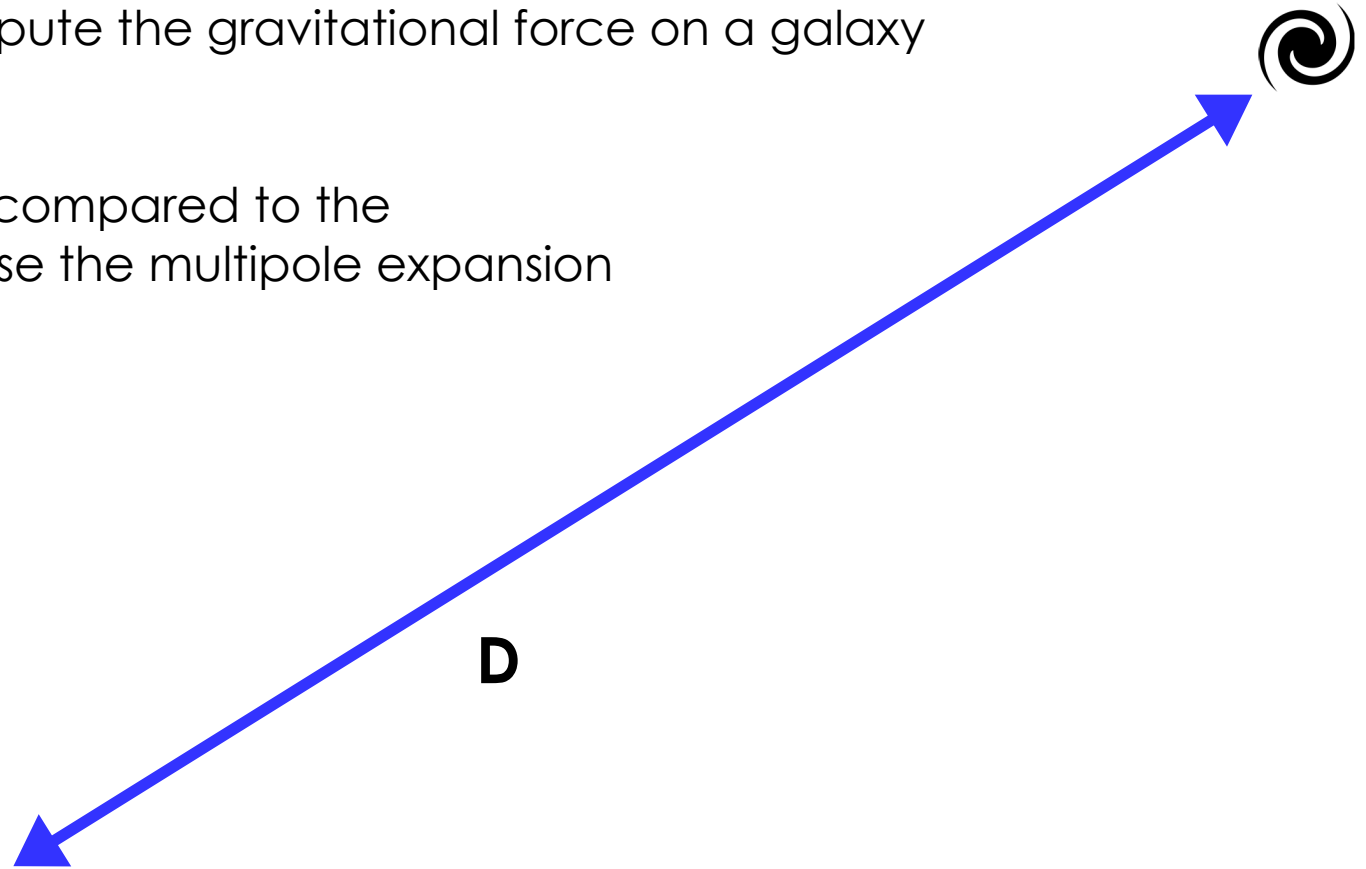
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L

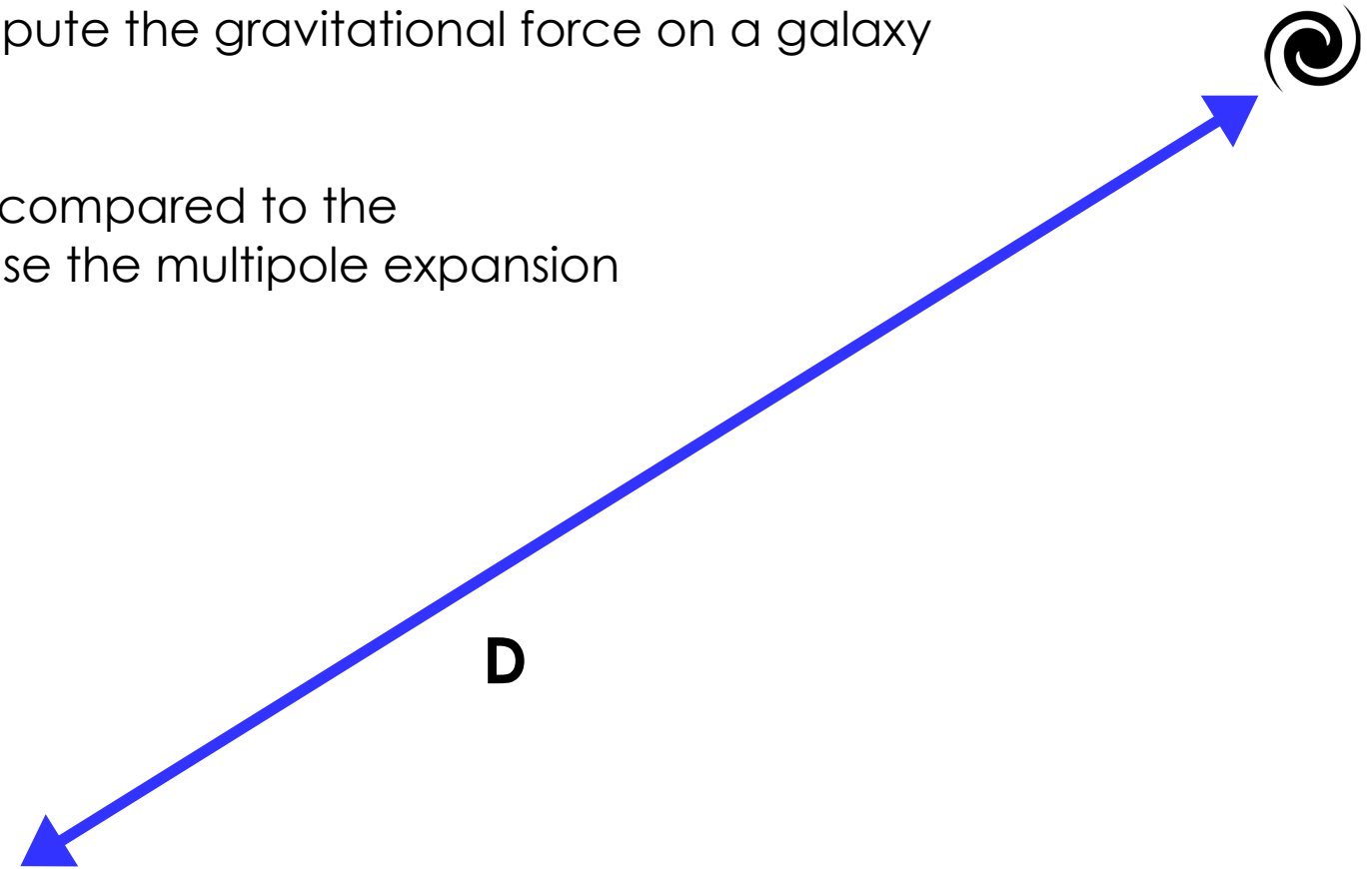
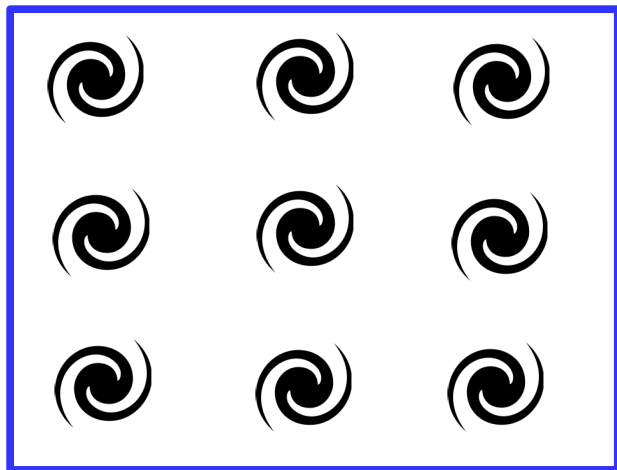


D

A zero bias field

Suppose we want to compute the gravitational force on a galaxy

At very large distances D compared to the size of the box L we can use the multipole expansion



If the distribution is spherically symmetric and the mean density is zero the galaxy far away will not feel any gravitational attraction.

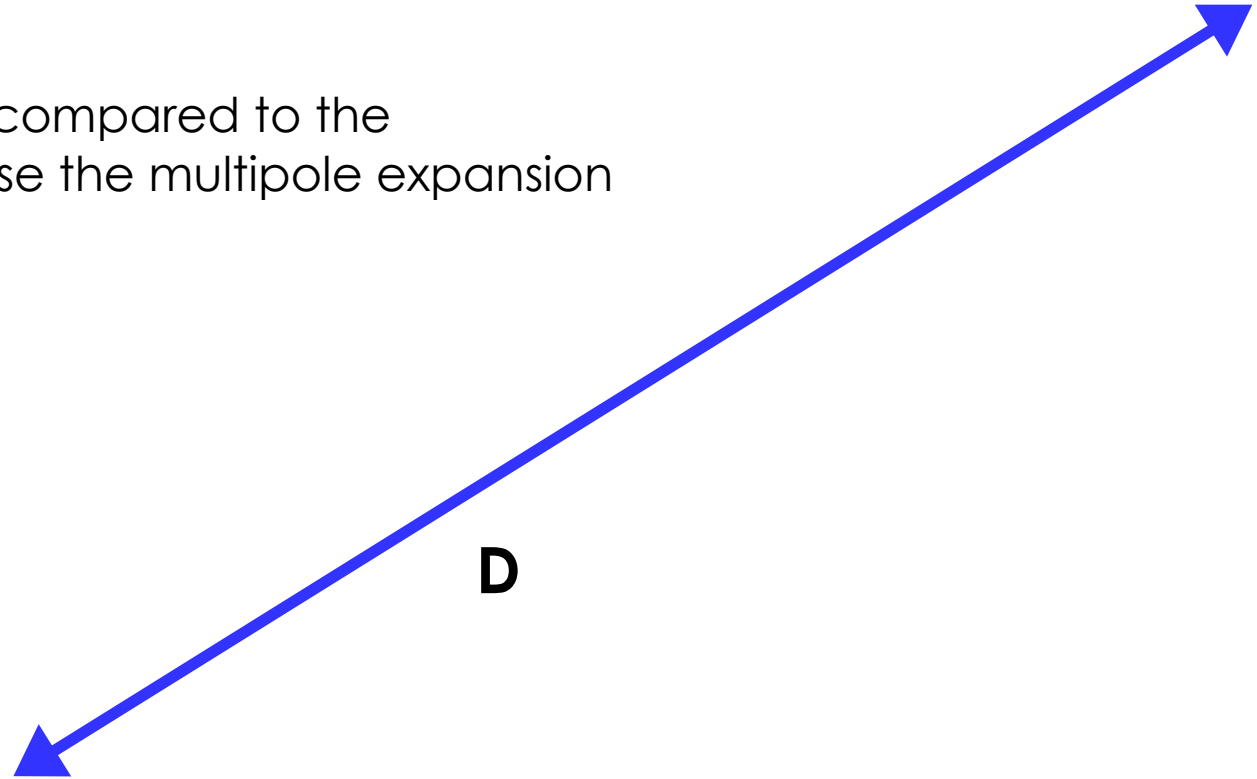
A zero bias field

Suppose we want to compute the gravitational force on a galaxy

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$$\Delta = 0$$

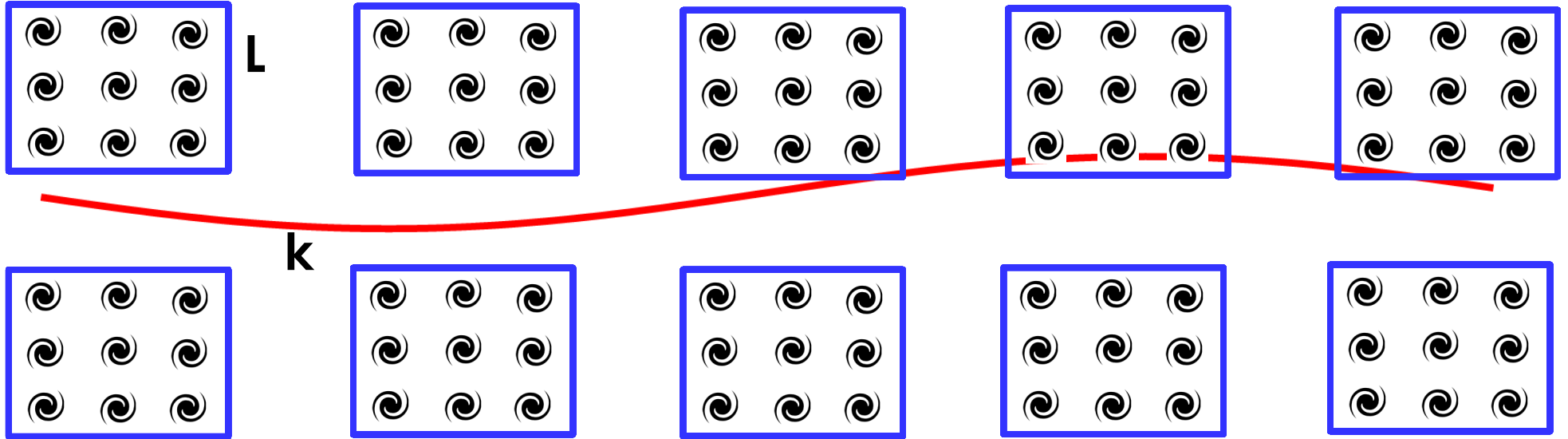


Empty !

L

If the distribution is spherically symmetric and the mean density is zero the galaxy far away will not feel any gravitational attraction.

A zero bias field

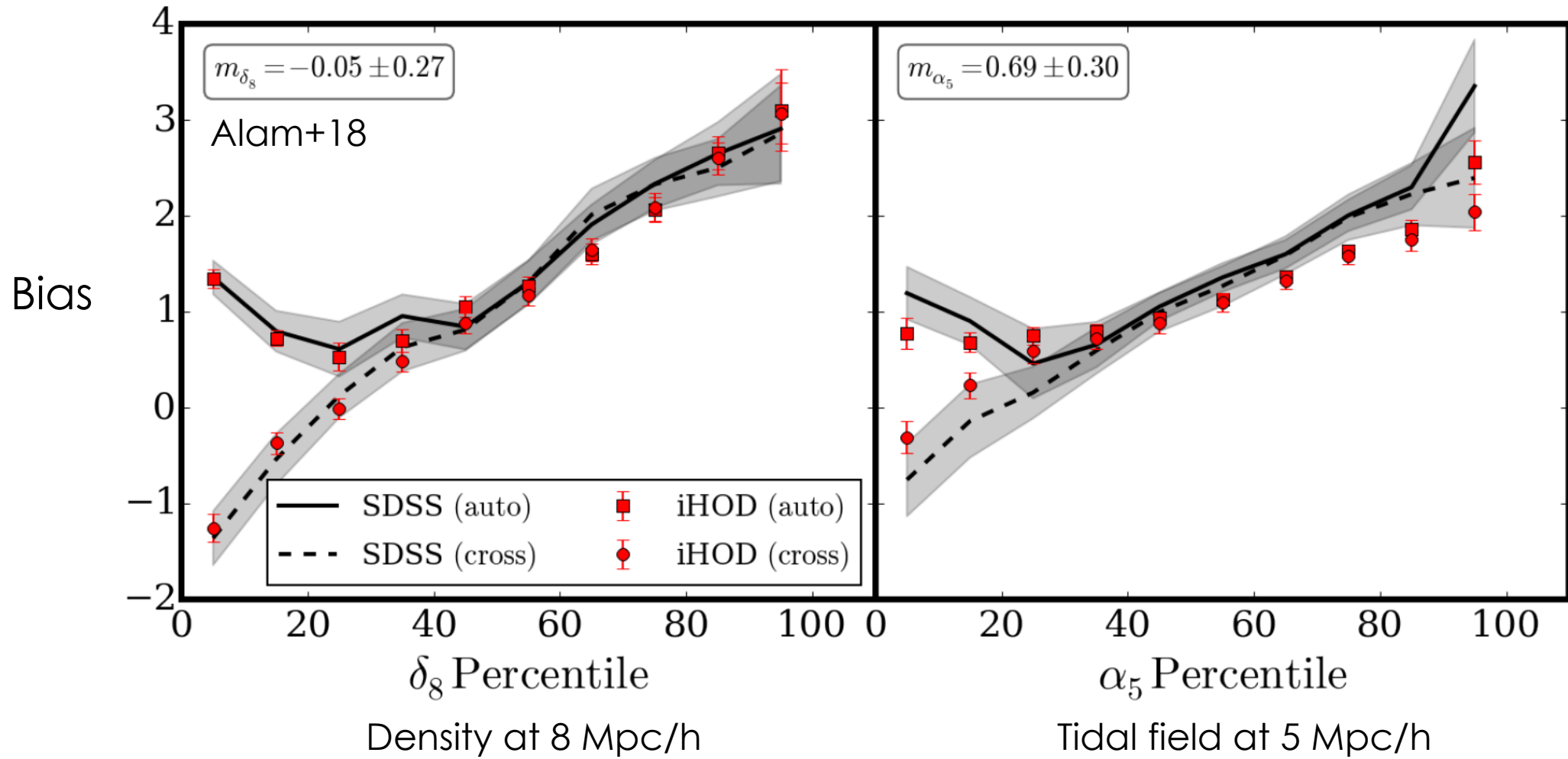


On scales much larger than L the power is zero

$$P_{\text{vortex}}(k \ll 1/L) \simeq 0$$

Complete understanding of this effect in Excursions Sets/Peaks theory

In real data, Alam et al. and Paranjape et al. 2018



25 % of all galaxies in Sloan main sample have zero bias

Simulations

1) Fix environmental threshold $1 + \Delta$ Mpc/h.

2a) Select all the halos in regions with $\delta_E \geq \Delta$ and measure their bias

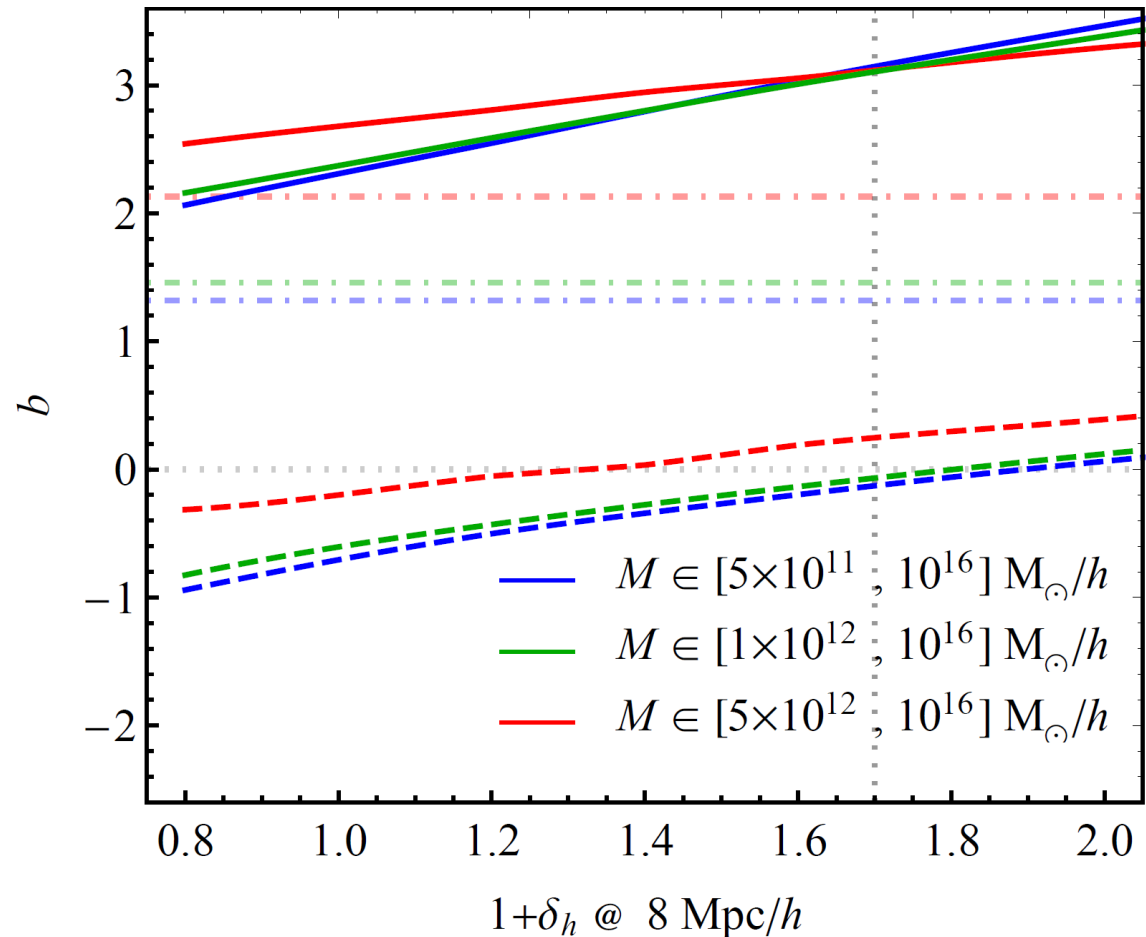
This is the *high bias* sample.

2b) Select all the halos in regions with $\delta_E < \Delta$ and measure their bias

This is the *low bias* sample.

25-40% of galaxies have zero bias

$1 + \delta \sim 1.7$ yields zero bias
and $b \sim 3$ for high bias sample.



Constraint on PNG

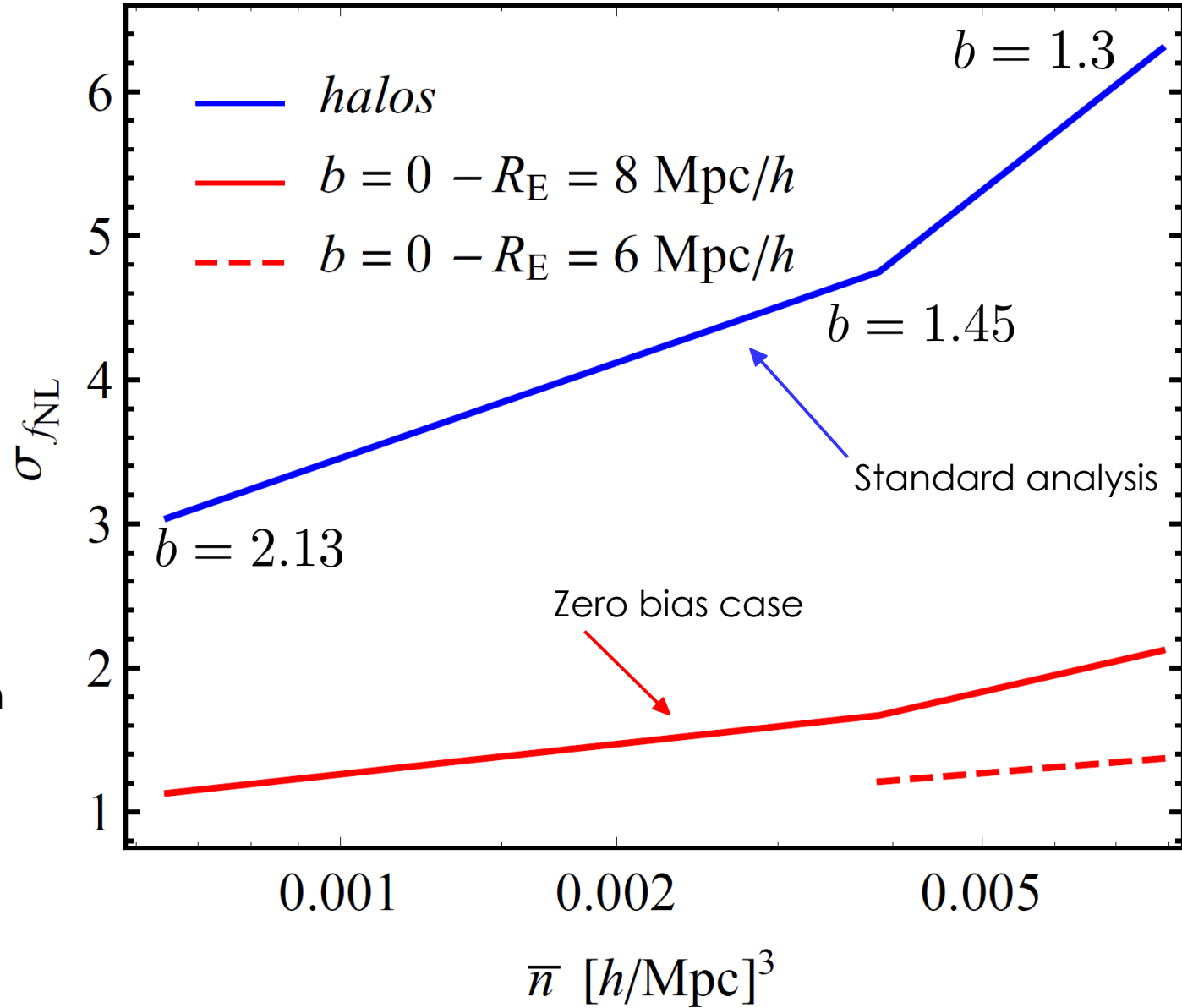
Setup:

- $z=1$.
- $V = 50 \text{ (Gpc/h)}^3$.
- Marginalized over other parameters.
- Non-Poissonian noise.
- Response from sims.

In the standard case no gain at high number densities.

In our approach 3x smaller error-bars.

Check with galaxy mocks!



Reality vs Fisherland

Even for BAO, the real data analysis never yields the Fisher numbers...

- Unaccounted sys, modeling issues, etc...

Our analysis is never optimal

- We never do the full C^{-1} on the data.

At high k , for Gaussian fields with \sim uniform noise, FKP is optimal for band-powers

Tegmark+98

- We never do optimal signal weighing for cosmological parameters

E.g. Optimal estimator for fNL in CMB is not just measuring the bispectrum.

Creminelli+06

Zhu+14, pair weighing for BAO, Ruggeri+16 for RSD, Mueller+16 for fNL , eBOSS DR14

Drawback is that we need an idea of the z -evolution of the signal

Reality vs Fisherland

eBOSS DR14:

- 180k QSOs in $0.8 < z < 2.2$

Lots of other QSOs at $z < 0.5$ and $z > 2.2$

- $n(z) < 10^{-5} \text{ [Mpc/h]}^{-3}$

Noise dominated, $n_P \ll 1$ at any scale

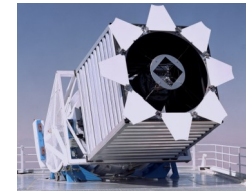
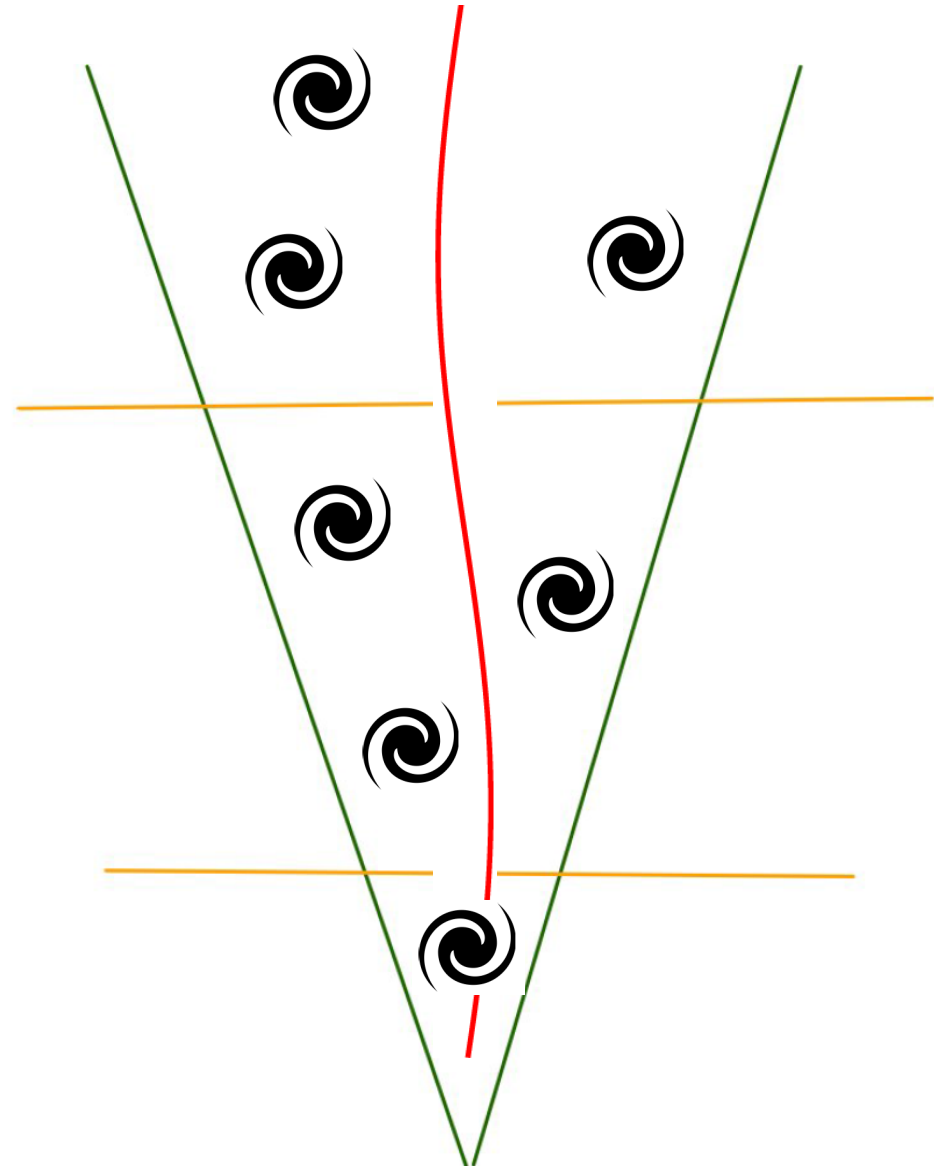
- 5% of the sky, $V \sim 10 \text{ [Gpc/h]}^3$

- Still contamination at low- k

Ideal testbed for f_{NL} analyses.

Redshift binning destroys info along LOS,
1/3 of the modes relevant for f_{NL} .

Full volume analysis + optimal weights.



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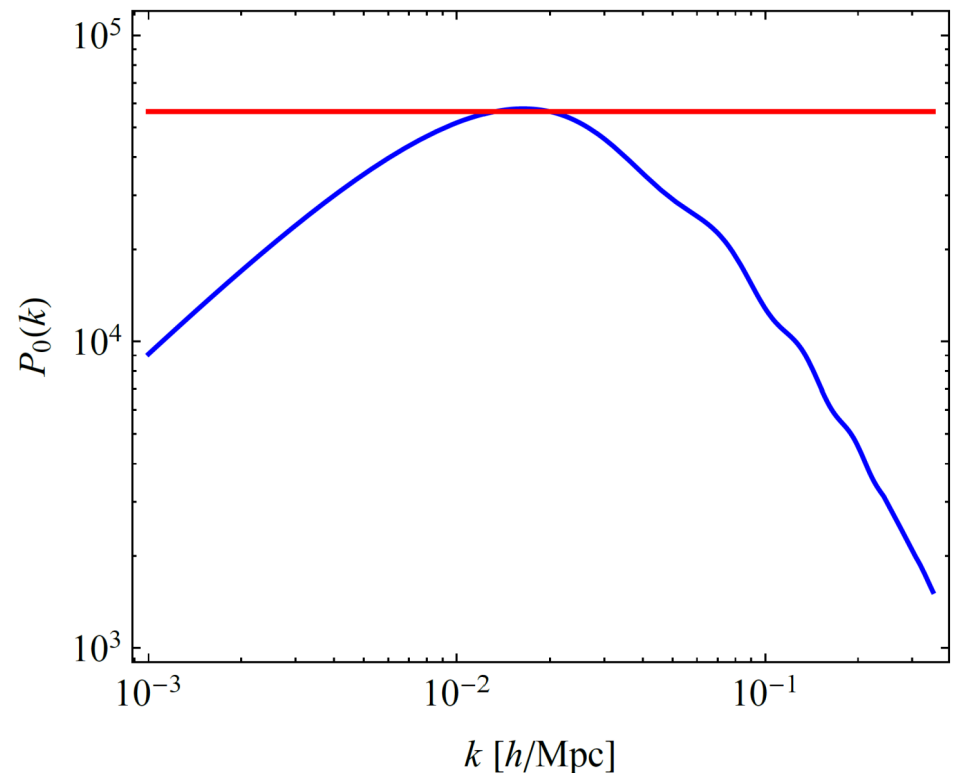
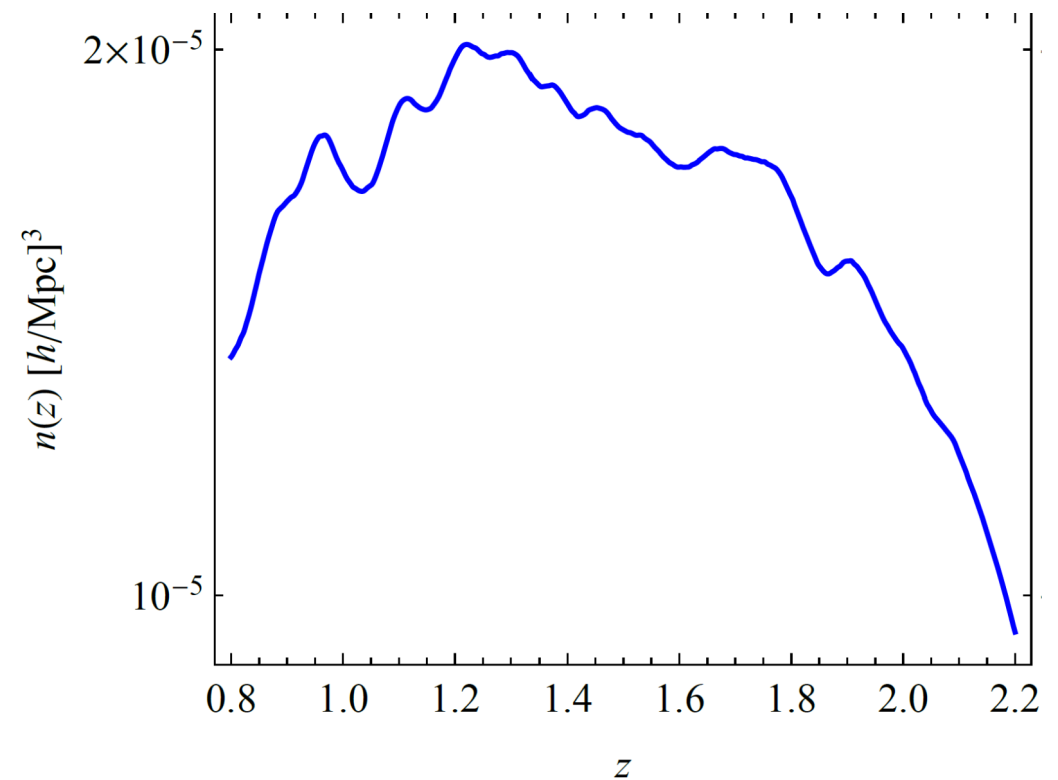
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 $\sim 1/3$ of the modes relevant for fNL.

Full volume analysis + optimal weights.



Reality vs Fisherland

$$P_{gg}(k, \mu, z) = [b(z) + f(z)\mu^2 + f_{NL}(b - 1.6)\alpha(k, z)]^2 P(k, z) + \frac{1}{\bar{n}(z)}$$

- (b-1.6) is more appropriate for QSOs? Wrong weighing leads to worse errorbars

If I evaluate the model at z_{eff} given by FKP weights

$$F_{ij}^{\text{eff}} = \frac{1}{2} V \int_{k_{\text{min}}}^{k_{\text{max}}} \frac{d^3 k}{(2\pi)^3} \frac{\partial P_{\text{qso}}(k, \mu; z_{\text{eff}}) / \partial \theta_i}{P_{\text{qso}}(k, \mu; z_{\text{eff}}) + n_{\text{qso}}^{-1}(z_{\text{eff}})} \frac{\partial P_{\text{qso}}(k, \mu; z_{\text{eff}}) / \partial \theta_j}{P_{\text{qso}}(k, \mu; z_{\text{eff}}) + n_{\text{qso}}^{-1}(z_{\text{eff}})}$$

The optimal analysis integrates over redshift

$$F_{ij} = \frac{1}{2} V \int_{z_{\text{min}}}^{z_{\text{max}}} dz \int_{k_{\text{min}}}^{k_{\text{max}}} \frac{d^3 k}{(2\pi)^3} \frac{\partial P_{\text{qso}}(k, \mu; z) / \partial \theta_i}{P_{\text{qso}}(k, \mu; z) + n_{\text{qso}}^{-1}(z)} \frac{\partial P_{\text{qso}}(k, \mu; z) / \partial \theta_j}{P_{\text{qso}}(k, \mu; z) + n_{\text{qso}}^{-1}(z)}$$

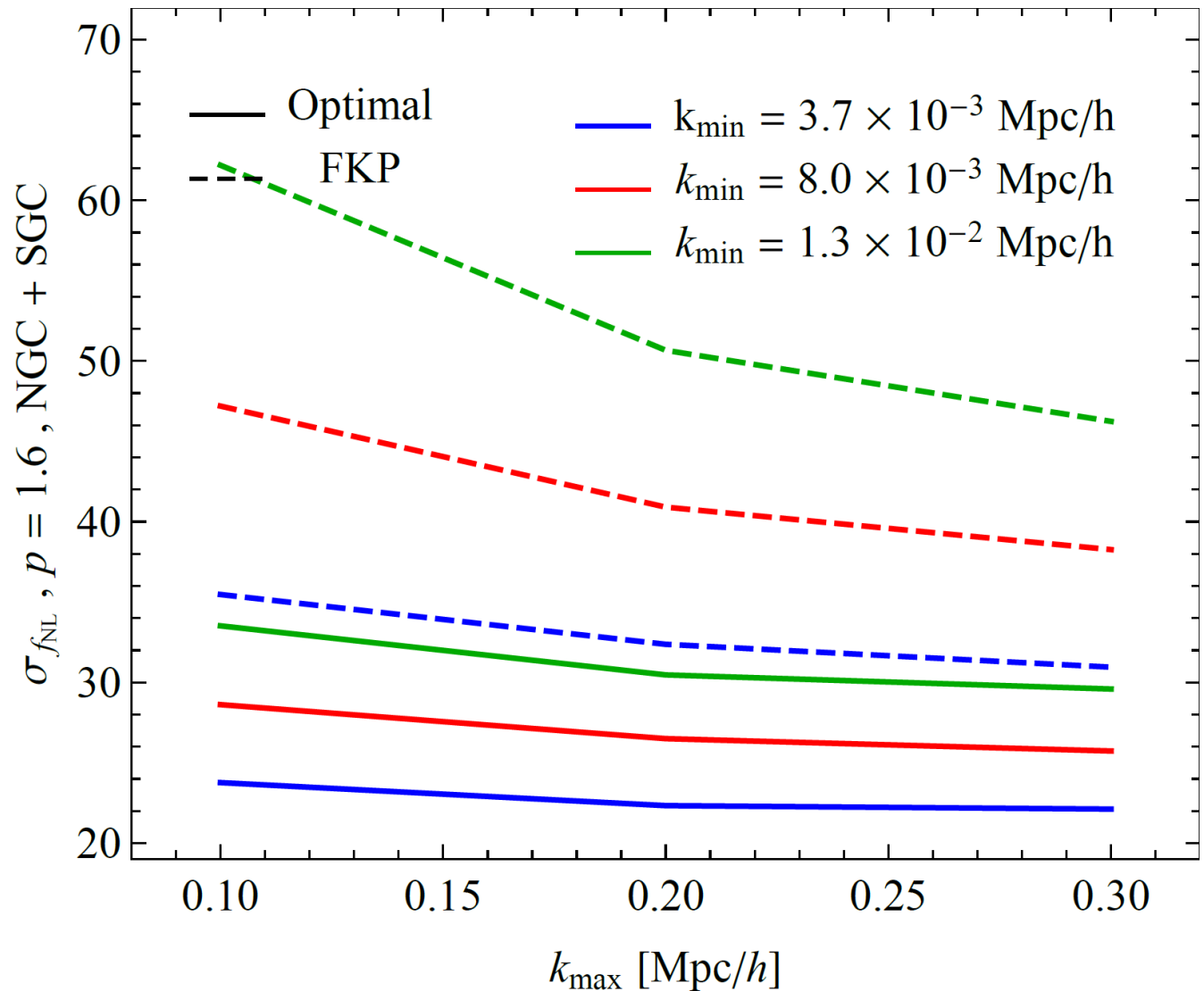
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A standard analysis
throws away 30-50% of the
signal on PNG.
Even in Fisherland.

Crucial to add redshift
evolution!
The right mocks???

How do we include it
in the data analysis?



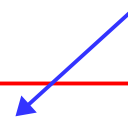
OQE

An optimal quadratic estimator is the answer. Given a set of galaxies positions

$$\hat{q}_{f_{\text{NL}}} = \sum_{\mathbf{x}_1, \mathbf{x}_2} \frac{1}{2} \frac{\delta(\mathbf{x}_1)}{C} C_{,f_{\text{NL}}} \frac{\delta(\mathbf{x}_2)}{C}$$

It is easy to show that

Multipoles estimator



$$\hat{q}_{f_{\text{NL}}} = \int \frac{dk k^2}{2\pi^2} P_m(k, z_0) \alpha_0(k) \int \frac{d\Omega_k}{4\pi} [\delta_0^{\tilde{w}}(-\mathbf{k}) \sum_{\ell=0,2} \delta_\ell^{w_\ell}(\mathbf{k})]$$

$$\tilde{w}(z) = b(z) - p \quad , \quad w_0(z) = D(z)(b(z) + f(z)/3) \quad , \quad w_2(z) = D(z)f(z) \quad .$$

Optimally cross correlation of weighted fields.

Upweights high redshift objects, where fNL response is the largest.

OQE

$$\tilde{w}(z) = b(z) - p \quad , \quad w_0(z) = D(z)(b(z) + f(z)/3) \quad , \quad w_2(z) = D(z)f(z) \quad .$$

No need to take absolute value and sqrt() of pair weights. Same solution in $\xi(r)$ or $P(k)$.

Similarly to FKP weights, they "move" the survey up and down. No integrals over z .

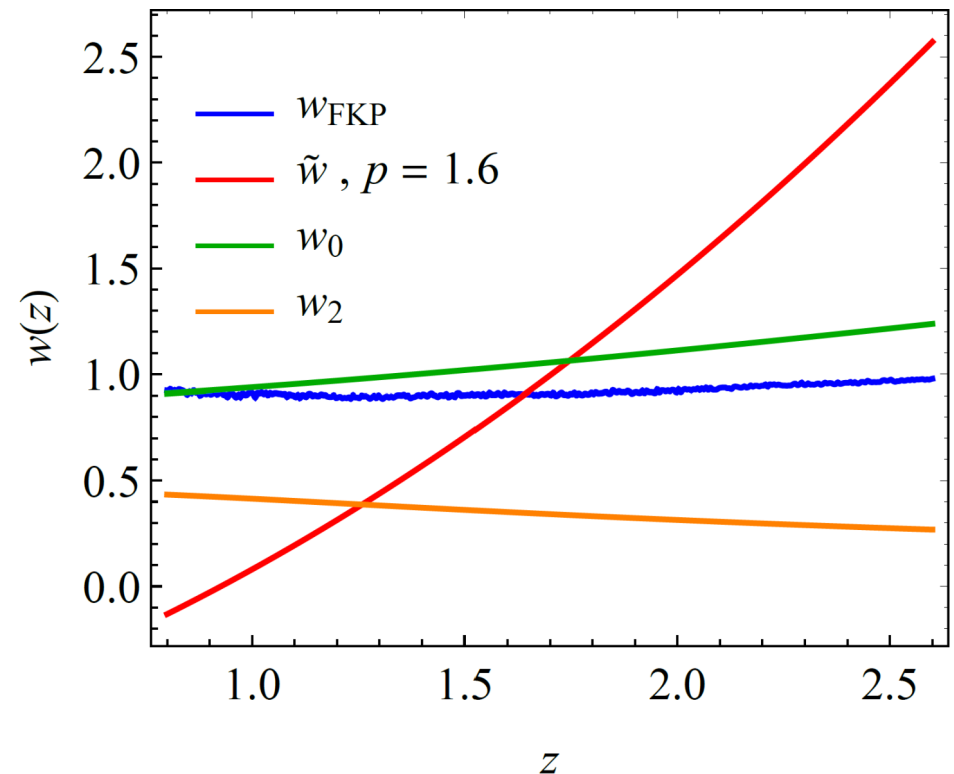
Optimal analysis boils down to a redefinition of the effective redshift.

FKP weights:

$$z_{\text{eff}}^{\text{FKP}} = 1.55$$

In the optimal case:

$$z_{0,\text{eff}} = 1.82 \quad , \quad z_{2,\text{eff}} = 1.79$$



Conclusions

Understanding the statistics of the initial conditions of the Universe is still an open issue.

Limited by sample variance, ie statistical errors.

A new result:

- Cosmic variance cancellation with 1 single tracer with zero bias.

- 3X improvement over standard analysis for Euclid/DESI numbers.

Check with realistic mocks

A new method:

- Optimal redshift weighing by the fNL response.

- 30-50 % improvement over standard methods.

- 5% of the sky and $n \sim 10^{-5}$ is competitive with CMB!

Move our efforts to the systematics at low k . Change of paradigm?

OQE

Optimal analysis boils down to a redefinition of the effective redshift.

$$P_{A,eff}(k) = (2A + 1) \int \frac{d\Omega_k}{4\pi} d^3 s_1 d^3 s_2 e^{i\vec{k}(\vec{s}_2 - \vec{s}_1)} \delta(\vec{s}_1) \delta(\vec{s}_2) W(\vec{s}_1) W(\vec{s}_2) \mathcal{L}_A(\hat{k} \cdot \hat{s}_1) \quad (1)$$

$$= (-i)^A (2A + 1) \sum_{\ell, L} \begin{pmatrix} \ell & L & A \\ 0 & 0 & 0 \end{pmatrix}^2 \int ds s^2 j_A(ks) \int ds_1 s_1^2 \xi_\ell(s; s_1(z)) Q_L(s; s_1(z)) \quad (2)$$

Or

$$P_A(k; z_{\text{eff}}) = (-i)^A (2A + 1) \sum_{\ell, L} \begin{pmatrix} \ell & L & A \\ 0 & 0 & 0 \end{pmatrix}^2 \int ds s^2 j_A(ks) \xi_\ell(s; z_{\text{eff}}) Q_L(s)$$

OQE

Optimal analysis boils down to a redefinition of the effective redshift.

Effective redshift approximation is 1% accurate
In $0.8 < z < 2.2$.

For FKP

$$z_{\text{eff}}^{\text{FKP}} = 1.55$$

In the optimal case:

$$z_{0,\text{eff}} = 1.82, \quad z_{2,\text{eff}} = 1.79$$

